NECESSITY, POSSIBLE WORLDS AND ONE AXIOM OF THE ONTOLOGICAL ARGUMENT

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1 Introduction

There has not been many subjects or problems in the history of science and, more generally, in the history of human spirituality that could be discussed from theological, philosophical and mathematical standpoint simultaneously. The so-called Ontological Argument of God's existence is such a subject. It is nowadays widely accepted that this problem originated in the 11th century in the field of theology.[1] It has described one property of God: existence. However, it soon became clear, based on the role that the terms *argument* and *existence* play in the Argument itself, which it could not have remained an exclusively theological subject. It has changed and assumed various forms throughout centuries. During a great part of this historical development, logical devices have been used in a rather informal manner.[2] It was only in the second half of the 20th century that, thanks to Gödel, the axiomatization of the Argument was made by means of the devices of the contemporary logic. Since then, it has been possible to observe it as a special, brief theory that has its own basic notions, axioms and theorems.[3] The final theorem of the Argument, which expresses its essence, states that out of the possibility comes also the necessity of God's existence.

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[1] See, for example, Dombrowski (2006)

[2] *Reductio ad absurdum*.

[3] *Axiom* 1 The conjunction of any number (collection) of positive properties is a positive property.

Axiom 2 A property is positive if and only if its negation is not positive: every property or (exclusive) its negation is positive.

Definition 1 *G(x)* or an object *x* is God-like if and only if *x* possesses all positive properties: for every property *A*, if *P(A)* then *A(x).*

Definition 2 A property *A* is an essence of an object *x* if and only if

1) *A(x)* and
2) for every property *B* of *x,* necessarily every object *y* which has the property *A* has the property *B* too.

Axiom 3 If a property is positive (or negative), it is necessarily positive (or negative).

Theorem 1 If *x* is God-like, then the property of being God-like is an essence of *x:* if *G(x),* then G is an essence of *x.*

Definition 3 Necessary existence. *E(x)* if and only if, for every essence *A* of *x,* there exists necessarily some object which has the property *A.*

Axiom 4 P(E). The property of necessary existence is a positive property.

Theorem 2 If *G(x),* then necessarily there is some object *y, G(y).*

Considering the conceptual apparatus of the Argument, it can be observed that definitions and axioms include one primitive concept (*a positive property*), two defined concepts (*God*, *necessary existence*), and the modal operator of necessity. Regardless of any particular axiomatization, it is only natural to ask questions related to the manner in which it was selected. By which criteria and by which idea was its creator guided in the process of introducing the primitive and defined concepts and while writing each axiom? This text will not deal with the general questions of the formal and non-formal conditions expected in every axiomatization[4], but will focus on the intuitive background of one of the Gödel's axioms:

If a property is positive (or negative), it is necessarily positive (or negative).

There are minimum two reasons why this particular axiom deserves special attention. Firstly, from the viewpoint of the Argument's chronology, this is the first axiom in which the operator of necessity was introduced, even though "through the back door". By this, we mean that Gödel had never said explicitly anything about the language he used in the Argument, although it is generally accepted that he had in mind language S_5 of the modal logic. Secondly, based on the additional explanations that can be found in the Gödel's original notebook[5], it is evident that this axiom and the notions mentioned in it have a special status. Namely, as the notes on the margins have shown, Gödel felt that he should explain them further using non-formal commentary. Generally speaking, it is clear that these comments contribute to a better understanding of the axiomatization and its resulting theory, but it must also be noted that they are not common practice of the authors who create axiomatizations. In these spots in the text, Gödel ceases to be exclusively a mathematician who abides strictly and only by the imperative of the logical precision and fulfilment of the formal conditions of the theory which he axiomatizes. He transgresses the border of the strict

Theorem 3 If it is possible that there is some God-like object, then it is necessary that there is some God-like object.
Axiom 5 If *A* is a positive property and [if] necessarily for all *x, A(x)* necessarily implies *B(x),* then B is a positive property. Wang (1996, pp. 114-115).

[4] When we speak about the basic *formal conditions* fulfilled by a set of axioms in a theory, we mean *consistency,* completeness, independence. As examples of the non-formal conditions, we can mention usefulness and productivity of the axioms.

[5] Gödel (1995, pp. 403-404). Today we know that there are two pages of the original Gödel's notebook that dates back from the 10.2.1970.

formalism and describes the meaning of the primitive concepts that he uses. This situation, mathematically unconventional as it is, represents a good stimulus to say something about necessity in general, as well as something about the manner it has been used in the Axiom. Unsurprisingly, we can expect a direct objection to the choice of the topic from relentless proponents of formalism. We could be asked: what is the purpose of discussion about the meaning of axioms, or undefined notions, apart from their formal role within the system? Or, is there any sense in discussion of anything except of the formal conditions, for example, of the independence of the axioms, etc.? Gödel seems to have already answered these questions by providing additional explanations which by no means reduce the formal value of the theory. Those explanations emphasize the importance of the intuitive meaning of the basic notions and axioms of the Argument.

2 Intuition of the Notion of Necessity

Interest in modal notions can be found in ancient and scholastic manuscripts.[6] Let us mention the *Prior Analytics,* where we can read the following:

Since there is a difference according as something belongs, necessarily belongs, or may belong to something else (for many things belong indeed, but not necessarily, others neither necessarily nor indeed at all, but it is possible for them to belong), it is clear that there will be different syllogisms to prove each of these relations, and syllogisms with differently related terms, one syllogism concluding from what is necessary, another from what is, a third from what is possible.[7]

In addition, in valid forms of Aristotle's categorical syllogism, we can see that there exists a certain kind of modal relationship between the premises and the conclusion. We say that an argument is valid if, by means of it, it is *not possible* to obtain a false proposition out of a true one. This relation can be expressed in other words: truthfulness of a conclusion is necessarily derived from the truthfulness of the premises or, simply, the conclusion is necessarily derived from the premises. That connection lies on the foundation of the so called strict implication, about which Clarence Lewis wrote at the beginning of the 20th century.[8] *A* strictly implies *B,* if it is *impossible* to be *A* and ¬*B*. The notions of impossible, possible and necessary, along with

6 Knuuttila (1982).

7 *Prior Analytics* i. 8 ($29^{b}29$ - 35) in Ross (1928).

8 Lewis C. I. (1912, pp. 522-31).

the strict implication, formed an intuitive basis upon which he constructed the five axiomatic systems for propositional modal logic — the so-called S_1–S_5.

The operator of necessity is a part of the contemporary modal logic's apparatus and, in the contemporary reference books, in the axiomatizations of the modal logic's language, the operator of necessity (□) is taken as primitive, whereas the operator of possibility (◊) is found as defined (◊ ↔ ¬ □ ¬).[9] In that case, the operator of necessity is introduced as a primary device without any additional explanation of its meaning. Formally, there is no obligation to explain it, given that it is a primitive notion. However, there are authors who do not want to leave it entirely without an elementary explanation of its intuitive background. In most cases, they would mention that, regardless of the fact that it is a basic notion, in the modal system Kripke's explanation is implied, according to which a proposition is necessary if and only if it is true *in all accessible possible worlds*.[10] More cautious authors, nevertheless, would be understandably diffident in using such a formulation. It is not only for the reasons of a metaphysically vague undertone contained it, but also because Kripke himself did not see any purpose in defining it more precisely when he first used it.[11] The insufficient clarity of the notion of possible worlds inspired various philosophical interpretations, and this is not all. Some authors have felt that they could question the sense of the basic modal notions and, consequently, the sense of the systems constructed on those

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[9] Sometimes we can find several axiomatizations for one theory. If for one axiomatization of a theory T, A_1 represents a set of basic propositions (axioms) and B_1 a set of deduced propositions, and for another axiomatization of the same theory T, A_2 represents a set of basic propositions (axioms) and B_2 is a set of deduced propositions, then $A_1 \neq A_2$, and $B_1 \neq B_2$, because this is the case of different axiomatizations. However, since these are axiomatizations of one and same theory, then $A_1 \cup B_1 = A_2 \cup B_2$. In other words, a set of all truths made of sets of all basic and derived propositions is the same in both axiomatizations, but the basic proposition in the first axiomatization can be the derived proposition in the second, and the basic proposition in the second can be the derived in the first axiomatization. For further information, see, for example, Tarski (1994, pp. 112, 113, 131).

A similar consideration can be applied to the notions used in theory. The primitive concepts of one axiomatization of a theory are sometimes derived concepts of another axiomatization of the same theory, whereas the derived concepts of one axiomatization, can be the primitive concept of the second. 54

[10] Even though in this text the notions of necessity will be used in this sense, it should be said that there are many explanations of modal notions that have evaded the concept of possible worlds. For further information, see, for example, Vetter (2011), Jacobs (2010), Placek (2011).

[11] Kripke (1959, p. 2). Even the "Semantical Analysis of Modal Logic", published four years later, does not say much more about the possible worlds.

notions.[12] Of course, we can say that the formal conditions for axiomatization of any mathematical theory are strictly defined, whereas this is not the case with the non-formal conditions. Namely, it is implied that in the foundation of every mathematical theory there exists an intuition about the objects described by that theory. There is, however, no set of precise conditions that would define the statement of that intuition. On the other hand, a more detailed description of this intuition would contribute significantly to a better understanding of the whole theory, as well as to the establishing its philosophical grounds.

Intuitively, the modal notions of *necessary* and *possible*, taken from the natural language, are related to the (un)conditional realization of a state of things or, equivalently, to the (un)conditional truthfulness of a claim of a state things. We can observe two equivalent characterizations: the modal qualifies the predicate, or the modal qualifies the truth of a claim. For instance, we can state:

It is necessary (possible) that Felix will jump two times from the verge of the Universe.

But, also

It is necessarily (possibly) true that Felix will jump two times from the verge of the Universe.

In order to make it simple, we will hereinafter use the first form of expression.

Even though, formally speaking, the notion of the *possible* can be defined by means of the notion of the *necessary*, the intuitive background of the former seems to be much clearer than that of the latter. Namely, when we speak about a *possible* realization of an event, we mean that under certain circumstances that event is realizable, but not under any circumstances. In other words, we think that there can be some circumstances that could prevent the realization of the event in question. It is easy to find examples for *possible* events. It is possible that one will get married this year; it is possible that it will rain in Moscow tomorrow; Great Britain may substitute the pound with the euro. On the other hand, the notion of *necessity*, regardless of the fact whether it is introduced as a primitive or derived concept, cannot be easily explained in general terms, nor is it easy to find examples for it. We understand a necessary event at an intuitive level only if we venture on explaining it at all, as an event that is unconditional in some sense, that is, if we think that there are no circumstances that could prevent it from happening. In the above formulation, the

[12] Quine (1961).

syntagm *in some sense* indicates that several difficulties are contained in the intuition that we have about this notion. Why *in some sense* and what sense is it? Why do we not simply say that it is an unconditional event? Primarily, it is probably because we suspect that there can be one single event in our lives that cannot be affected by any conditions and circumstances whatsoever. Secondly, we most probably tacitly acknowledge a hierarchy of conditions, of those conditions relevant for a consideration of necessity of a particular event and of those that are not relevant, or their relevance is almost insignificant. Finally, if we do not want to leave the beforehand mentioned theoretical argumentation indefinite and vague, we must attempt at providing an example of a necessary event that might clarify the intuition. However, this generates a problem. Namely, we usually choose those events that appear as constant in some way, immune to the human actions or natural impacts that sometimes determine the events in our world. For example, we chose events that are consequences of astronomical movements, such as: tomorrow's sunrise in San Sebastian, the beginning of spring in Calgary, the finding of Halley's Comet near the Earth, etc. Indeed, these events can hardly be influenced by any person's mood, or any government's decision, or an atmospheric motion in the Earth's atmosphere. However, it is clear that the realization of those events also depends on some conditions which are indisputably more stable and permanent that the conditions which determine the occurrence of majority of the events in our world, but they are *nonetheless* conditions. Needless to say, if we decide to accept that there are no absolutely unconditional events which could be regarded as exclusively necessary, then we are faced with a new obstacle. If all events are conditioned in one way or another, and if we assume that the domain of the operator of necessity is not empty, how shall we define which events are necessary and which are not? Are there events, states of things, facts that can be said to be necessary?

As an illustration of the necessary states of things, reference books often give mathematical contents. Let us consider the following facts as examples:

> *In a Dedekind plane, given two angles α and β, there exists an integer n such that the angle $\alpha/2^n$ is smaller than β;*
>
> *or*
>
> *There are infinitely many prime numbers.*

Why do we say that these are necessary facts and to what do they owe their necessity? In both instances, we arrive at the facts by evidence that does depend on any random and variable conditions.[13] Therefore, the evidence is always deducible in the same way only if the *stipulated conditions* are fulfilled, that is, the conditions that are *implied* whenever we describe a specific situation/state of things. In other words, that state of things will always exist if we speak about those objects, since the mere mentioning entails the existence of certain conditions. In the first example offered, those conditions refer to the properties of the Dedekind's plane, to the properties of every angle given in that plane, to the property of the relation *smaller* in the set of all angles in the plane, etc. In the second example, those conditions refer to the properties of the prime numbers, to the existence of definition of the infinite set, etc. The following example, taken from geometry, indicates very clearly to what extent the necessity in mathematics depends on the stipulated conditions related to the described state of things. Let us consider this proposition:

The sum of the angles of a triangle is always smaller than two right angles.

We can state that the described state is necessary, but under the certain *stipulated* conditions, namely, solely in the case of the non-Euclidean geometry. However, if the stipulated conditions are different, if, for instance, it is a matter of the Euclidean geometry, we cannot speak about the necessary state of things, but, moreover, that state of things is impossible.

The abovementioned illustrations have been offered as the examples of the state of things that we regard as intuitively necessary. Nevertheless, how should we treat the areas that cannot be exactly described? Can we also apply mathematical models in those cases? In order to at least suggest an answer to this question, in accordance with Kripke's definition of necessity, it is essential to offer first a brief reminder of the notion of *possible worlds*.

3. Three Ideas of Possible Worlds

The question of possible worlds is today considered within numerous contemporary theories that oscillate between philosophy and mathematics.[14] Only several elementary attitudes, drawn from three traditional theories and useful for this discussion, will be mentioned here. The classical reference regarding this topic is the book titled *Counterfactuals* by David Lewis. In this book, we find faith in the existence of

[13] Borceux (2014, pp. 90, 338).

[14] See, for example, Yagisawa (2010), Girle (2003), Divers (2002).

possible worlds different from our world, and in these worlds, the things can be different than we know them. Thus, the initial definition would be: "possible worlds" = "ways things could have been".[15] Based on this definition, we can create an image of the possible worlds as some kind of hypothetical situations in which we can imagine counterparts of things that belong to our world. We do not see any realistic aspirations in that definition because of which we would think about some physically and temporally different worlds. The passages that follow, however, bring in certain clarity:

...When I profess realism about possible worlds, I mean to be taken literally. Possible worlds are what they are, and not some other thing. If asked what sort of thing they are, I cannot give the kind of reply my questioner probably expects that is, a proposal to reduce possible worlds to something else.

I can only ask him to admit that he knows what sort of thing our actual world is, and then explain that other worlds are more things of *that* sort, differing not in kind but only in what goes on at them. Our actual world is only one world among others. We call it alone actual not because it differs in kind from all the rest but because it is the world, we inhabit. The inhabitants of other worlds may truly call their own world a actual, if they mean by "actual" what we do...

...Our present time is only one time among others. We call it alone present not because it differs in kind from all the rest, but because it is the time, we inhabit. The inhabitants of other times may truly call their own times "present"... [16]

Two details, among others, can be deduced from the above quotation. First, Lewis considers the notion of possible worlds as *primitive*, that is, he does not feel that it should be defined by more elementary notions. The most he can state about them is that they are the "ways things could have been". Secondly, a realistic mode of description is indisputable. Those worlds, just as our actual world, have (had, will have) their own coordinates in space and time. These not unreal abstractions or empty speculations would be impracticable in reality. Realism, in this context, is not conceived as a mere description of *our* reality, the *here* and *now*, but as a description of *any* reality *anywhere* and *anytime*. More precisely, using the terminology of analytic geometry, our reality is coordinate origin, and a sum of possible worlds, according to Lewis, is an entire coordinate plane of two-dimensional coordinate system whose coordinate axes

[15] Lewis D. (2001, p. 84).

[16] Lewis D. (2001, pp. 85-86.).

are space and time. [17] Each of the possible worlds, that is, an arbitrary point in the coordinate plane, is an entity of the same kind as the point of coordinate origin. All those entities differ from each other only by the fact that different coordinates describe them. The existence of entities is not a product of imagination of an inhabitant of the actual world, since, if such were the case, that inhabitant would be able to tell us all about them. [18] We could rather say that the faith in their existence is a natural attitude of any realist who is not infected by "geocentrism".

The first impression can be that this point of view on possible worlds is rather vulgar and materialized. However, there is a more extreme – Quine's viewpoint, which might have been inspired by a lack of precise details about possible worlds, as well as by a desire for the so-called "ontological economy". [19] This is the utmost simplification of the image of possible world, based on the assumption that this world is Euclidean space composed of a homogeneous matter. It is, in effect, an explicit suggestion that a mathematical model can represent an arbitrary possible world. Quine's idea can be paraphrased in a few stages. First, every point in space can be filled either with a matter or with empty. Second, granted that an arbitrary point of an arbitrary world can be expressed by the ordered number triples of real numbers, the state of things in that point can then also be expressed by that object. Third, since the Euclidean space is composed of points, it ensures that the entire possible world can be represented as a sum of all possible number triples. Fourth, as the nature of a world should not depend on the manner in which the coordinate axes are set, nor on the basic unit of measurement, it follows that all the worlds- Euclidean spaces whose coordinate systems can be copied one into another by rotation or homothety can be regarded as one and same possible world. Fifth, the last coordinate axis – the one that represents time – has been introduced into the system. Thus, we arrive at a possible description of a possible world:

A possible world finally can be explained in somewhat the same way but with four dimensions, representing space-time. A possible world becomes, roughly, any class

[17] For the sake of simplicity, we can here assume that a specific space can be "coded" by a real number from a real axis.

[18] Lewis D. (2001, p. 88).

[19] Quine (1969, p. 149).

whose members are all the classes that are geometrically similar to some one class of number quadruples.[20]

On the one hand, this mathematically structured image of a possible world could have three very practical consequences. Namely, if there is a sense in making a mathematical model that could represent a possible world in its most precise details, then, accordingly, we would be able to describe all the changes and processes in that world to the details. In addition, with that model we could avoid numerous risks of a subjective description of possible world, since such a description would be inevitably burdened with great arbitrariness. Eventually, as this approach would endow us with the exact image of any possible world, it would also provide a more precise comparison of the entities and their relations, as well as of the relation among various possible worlds. On the other hand, the attempt at mathematization carries certain difficulties. Let us first offer a technical supplement to the Quine's idea. Even if we embraced such a simplified image of the possible world, it would be important to notice that, indeed, every point in space can be represented by the ordered number triples of the real numbers, but that this is insufficient description of the state of things. Namely, in accordance with this conception, at the elementary level of space, we observe the state of things as either a presence or absence of matter in an arbitrary point on space. In order to describe that fact (presence or absence) in the specific point in space, we need another coordinate apart from the existing three that determine the point's position. That additional coordinate would have, in an informatical manner, only two discrete values (for example, 0 and 1) on the basis of which we could differentiate between two discrete cases, the presence and the absence of the matter. Thus, by the ordered quadruples, the state of things in the point whose position is defined by the first three numbers would be doubtlessly established. With the additional axis, which would describe also the flow of time, we could say that, in the spirit of the Quine's idea, the possible world would be describable by means of the ordered quintuples class. Nevertheless, along with the suggestion for improvement of the idea, it is even more important to underline the objections that resist easy answers. At this point, we will not deal with the questions such as: why should we regard the possible world as an Euclidean space, or, is not the Democritus' perspective on the physical worlds outdated, or, what does mathematical notion of the point imply in a physical sense, etc. Even if

[20] Quine (1969, p. 151).

the mentioned questions were soluble, it is doubtful that the offered model could represent *any* state of things. We would be able to represent, for instance, Gavrilo Princip's assassination of Franz Ferdinand in 1914 in Sarajevo, or the building of Pyramid of Cheops in Giza, because it was about the "movement of the matter" through certain space within a certain period of time. All those bodies, whether alive or not, fill in or do not fill in certain points in space during certain times. Is there any way, however, to use this apparatus in order to describe such states of things as, for instance, the feeling of dejection that overwhelmed Napoleon while he was imprisoned on the St. Helena Island? Could we reply, in the spirit of the "ontological economy", that all that which occurs in the synapses of the specific beings can be eventually represented by either presence or absence of the matter and, subsequently, by an appropriate mathematical method? Obviously, we would encounter a problem. These questions could lead us towards contemplating the nature and properties of the point as the elementary entity in the suggested mathematical model, which would not be easy to handle.[21]

Finally, let us look at Kripke's understanding of the notion of possible worlds. His interpretation of this notion, which is significantly different from the two previously mentioned, is explained in *Naming and Necessity.* One will not find any trace of physical or vulgarly materialistic simplifications in this text. On the contrary, it can be said that Kripke was quite opposed to it. However, the beforehand mentioned simplifications were rather widespread at the time of the publication of *Naming and Necessity*, playing an important role in the philosophical circles. This made Kripke's mission more complex and more demanding. It was necessary to offer new explanations of the concept already uncomfortably overburdened with the influential current interpretations. It was, therefore, necessary to explain that it would be in vain to look for the possible worlds either on our own or on any other continent, as well as in our or in any other galaxy:

> "... the wrong way of looking at what a possible world is. One thinks, in this picture, of a possible world as if it were like a foreign country... A possible world is not a distant country that we are coming across, or

21 Let us be reminded that Euclid did not explain the notion of point any further in the *Elements* either, apart from saying that it is an object which has no smaller parts.

> viewing through a telescope. Generally speaking, another possible world is too far away. Even if we travel faster than light, we won't get to it…"[22]

This is an attempt, at the first place, to discourage, without using harsh words, all those who thought that it was possible to understand/search possible worlds by means of the high technology devices and their usage in crossing local, the Earth's or some larger distances. All the efforts to locate them anywhere on Earth or in the Universe will be aimless. They are not real spaces that exist in the same way as this space, which we inhabit. They make just a conception, an image of a possible outlook of our world. In other words, the very imagining of a possible world sets some of the conditions that partly describe it. The mere idea of a possible world brings in its conditions to our minds. What do we mean by saying that "in some other world I wouldn't have slipped on a banana peel today"? By this, we are imagining a situation in which the street I was passing along had not contained a banana peel or, if it had, I would not have stepped on it. We are not imagining a physical world describable by means of the "ontological economy" devices, the one in which we could analyse all the contents that compose it. It is only a *partial* state of things that could have happened in our world that we are imagining, only in the aspects related to the event with which we are dealing. We are interested only in the relevant details, which, in the above case, are only those that are connected to the fall, the act of slipping. Thus, the possible world cannot be imagined as a complete, realistically describable whole, but exclusively as a partial description of conditions related to a specific event.[23]

In the light of the above explanation, how should we understand Kripke's attitude that a proposition is necessary if and only if it is true in all accessible possible worlds? Or, analogously, how are we to understand that an event is necessary if it happens in all possible worlds? In order to reply to this question, it is indispensable and sufficient to observe that event/state of things in all possible worlds. By *all* we do not mean that, by using cardinal numbers theory and Quine's "ontological economy", we should go physically through all the theoretically and combinatorically possible worlds and check the state of things in which we are interested. In effect, by this we mean that we should establish a set of basic properties of the entities that make up that state of

[22] Kripke (2001, pp. 43-44).
[23] Kripke (2001, p. 44).

things, and which are the property of those entities in any possible world. In other words, we should have a criterion for their identification in all possible worlds in order to be able to state inherent and inevitable properties of the entities in any world, those that are *implied* whenever we speak about them. This idea is a natural one. Let us list all the inevitable properties of an entity with which we are dealing in order to see whether the state of things regarding them would be repeated in all other worlds. If such were the case, then that state of things is necessary, i.e. the proposition of it is necessarily true. It is, however, particularly difficult to determine the relevant properties. In relation to this, Kripke himself would not have been able to tell us much more about the relevant properties of Gavrilo Princip, for instance, apart from the statement that, doubtlessly, he is not a non-living being.[24] Namely, we have imagined him as a living being with some additional properties. Not all that which we have not beforehand implied by imagining him is necessary. This includes his committing of assassination, wearing a jacket, a missing button on the jacket, etc. Hence, we can say that a set of necessary properties that can be listed in such a situation is, generally speaking, rather limited and not easily definable.

> "... In the case of other types of objects, say people, material objects, things like that, has anyone given a set of necessary and sufficient conditions for identity across possible worlds?
> Really, adequate necessary and sufficient conditions for identity which do not beg the question are very rare in any case. Mathematics is the only case I really know of where they are given even within a possible world, to tell the truth. I don't know of such conditions for identity of material objects over time, or for people..."[25]

Therefore, we are explicitly warned that every attempt at identification of an entity through various possible worlds will raise some problems. The same outcome may be expected when we deal with necessity. Examination of necessity of a state of things is a considerably uncertain process, the result of which is utterly unpredictable. With the exception that, according to this instruction, a special position is reserved for mathematics. When Kripke says this, he probably has in mind the fact that all the properties of mathematical objects are either precisely stipulated or proved on the

[24] Kripke (2001, p. 46).
[25] Kripke (2001, p. 43).

basis of the determined ones, thus not depending on any variables as the non-mathematical objects do. For instance, the fact that 5 is a prime number is derived from the previously defined properties of the set of natural numbers, definition of divisibility and definition of the prime number. The number 5 is divisible only by 1 and by itself regardless of the variable circumstances in which we observe it: for example, as regular polyhedrons[26], or as a number of new-borns on a specific day in a specific town. Encouraged by this understanding, which endows mathematical objects with a privileged status, we return to the Ontological Argument.

4. How to understand necessity in the Axiom?

At approximately the same time when Kripke delivered his lectures on necessity and possible worlds, which later made part of *Naming and Necessity*, Gödel wrote down notes on the Ontological Argument.[27] As we noticed before, the operator of necessity appears for the first time in the third axiom. It tells us that every positive property is necessarily positive, as well as that every property, which is not positive, is necessarily non-positive:

$$P(\varphi) \supset N\,P(\varphi) \qquad \text{and} \qquad \sim P(\varphi) \supset N \sim P(\varphi)\,.$$

The beginning of the Argument offers explanations in notes. The letter φ stands for an arbitrary property, whereas the sign $P(\varphi)$ means that the property φ is positive. However, what is uncommon for axiomatizations is that, in addition to the Axiom, one can find here some explanations that are not related to the technical details whatsoever. Namely, there is a kind of *justification* of the Axiom, the explanations why the Axiom is what it is. Why do we say that it is an uncommon situation? It is only natural to expect that there must be an intuition behind every formalization; however, it rarely occurs that along with the axiom we find an additional interpretation of its meaning. The axioms that fulfill formal conditions are always regarded true and we do not expect any further explanations. The term axiom is synonymous with the unquestionably true statement, and not only in the specialists' circles. Informally speaking, we usually say that their truthfulness is based on their obviousness, whereas in mathematics the truthfulness is primarily regarded as a result of the formal conditions fulfillment. In this specific case, however, the explanation is not related to

[26] Borceux (2014, p. 106).

[27] Kripke held his lectures in January 1970 at the Princeton University, about twenty days before Gödel wrote his notebook.

the formal conditions. What we learn is that a positive property is necessarily positive, since *it results from the nature of the property*.[28] And what is that nature like?

Positive means positive in the moral aesthetic sense (independently of accidental structure of the world). Only then [are] the axioms true. It may also mean pure "attribution" as opposed to "privation" (or *containing* privation). This interpretation [supports a] simpler proof.[29]

Generally speaking, the author of an axiomatization creates a set of axioms and builds a theory upon it, respecting certain formal conditions. Usually, he has an intuitive idea about his creation, except when the axiomatization is derived from a simple combinatorics. In addition, in this process, there are no limitations regarding the use of different interpretations of the concepts in a theory if those interpretations are consistently applied. Indeed, it very rarely occurs that the author of an axiomatization suggests the *only* meaning of a notion that should be considered in order to make the axiom truthful. The above stated paragraph has done precisely this. Hence, the axioms of the Argument are not true in themselves, as we are accustomed to encounter in the formal theories. They are true only if the notions used in them are correctly interpreted. We have been provided with a precise instruction about what it means to *be positive*.[30] Unfortunately, Gödel is one of very few authors whose formal considerations have been followed by philosophical elaborations, either in the same or in separate texts. Regardless of the small number of researchers who follow his suit, it appears that we have been offered a good example of the fact that this kind of approach deepens and enforces the formalization. On the other hand, which other subject, if not the subject of God's existence, deserves something more than a mere formalism? Eventually, is this not the best approach for every axiomatization: to offer to the reader, not in a commanding manner, the meaning of notions, definitions and axioms which the author had in mind while creating the axiomatization?

Let us return to the last quoted paragraph. It contains rather modest instructions referring to the understanding of the positive property concept.[31] Can this

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[28] Gödel (1995, p. 403).

[29] Gödel (1995, p. 404).

[30] We could say that this is, in a certain sense, unprecedented example of creation of the formal theories – axioms become true only if their concepts are interpreted in one way, that is, only if one model, one interpretation is taken into consideration.

[31] *Positive means positive in the moral aesthetic sense* – this would certainly demand additional ethical and esthetical explanations.

be helpful in understanding why a positive property should *necessarily* be positive and how it is derived from nature of the feature? In this sense, it is useful to accept the attitude that positiveness does not depend on the *accidental structure of the world* or, to put it more daringly, that positiveness applies to *all possible worlds.* Are there grounds for this re-phrasing? There are certainly some authors who would dispute this interpretation.

Leibniz was Gödel's favourite philosopher. But one central idea of Leibniz's was never taken up by Gödel: the idea of possible worlds. This is one of the many indications of Gödel's actualism. He agreed for instance in so many words with Russell's statement that logic deals with the real world quite as much as zoology, albeit with its more abstract features.[32]

It is true that Gödel does not use the syntagm *possible worlds* explicitly anywhere in the Argument. Nevertheless, this concept is not unfamiliar to him, as he had used it before in his texts.[33] On the other hand, the necessity (the operator of necessity) used in the Argument, as we have already seen, can be quite naturally understood by means of the notion of possible worlds. We have already discussed some aspects of understanding that notion. Would it be possible to understand necessity in this specific case in the light of some of those aspects? In order to reply to this question, we need to make sure, if an arbitrary positive property, whatever this implies, is positive in all possible worlds. As we have already seen, Lewis had an ambition to represent possible worlds as actual spaces with definite location in space and time. According to him, these worlds are not subjective imagination of an individual, but a reality that can be discussed as a factual state. He did not say much, however, about a description of those worlds, nor about the modes of knowing them or defining their position. His idea of possible worlds generates unsurpassable obstacles to examining the actual state of things in one of those worlds. Namely, even if we neglect a great number of extremely sharp criticism directed, generally speaking, to the very meaningfulness of the idea of possible worlds as physically independent and existing entities,[34] we most certainly cannot ignore those critical comments that refer to the intelligibility and usability of

32 Hintikka (2000, pp. 47–48).

33 Gödel (1990, p. 206).

34 See, for instance, Powers (1976, p. 95).

Lewis's notion.[35] Indeed, Lewis himself added considerably to the incomprehensibility of the notion by demonstrating his inability to describe it in precise details. This unintelligibility is a direct result of the fact that we do not have unambiguous criteria for individuation and identity of the notion of possible worlds. We have not been given a set of basic properties because of which we could identify a particular possible world. Therefore, how can we make a comparison between various states of things in those worlds? More specifically, how can we contemplate the positiveness of a property in various worlds?

It is well known that there have been several attempts to defend Lewis's notion of possible worlds. According to one of them, individuation and identification of a concept are not necessary conditions for their understanding, since, for example, we cannot identify the concepts such as nation, war or personality, which are nevertheless clear and understandable. On the other hand, if we insist on these conditions, then it implies that they can be fulfilled by conceiving those worlds as equivalence classes of the primitive dyadic relation[36]: "is in the same world as".[37] However, these ideas contain considerable difficulties. Elimination of identification conditions, as they are main presupposition for understanding possible worlds, seems unacceptable, as the given example shows. Don't we have at least basic tools for identifying concepts such as nation, war or personality? To make it even simpler, can't we identify those concepts at least by means of ostensive definition, by pointing to their examples? Yet, there is no example of a possible world that we can identify in such a way, apart from our own world. Also, if we are to understand a specific possible world only as one of the equivalence classes formed by the mentioned equivalence relation, then we have another two problems in the consideration of necessity. First, every relation, the equivalence relation included, is defined upon a set, and equivalence classes are only subsets of that set. Thus, in accordance with this, possible worlds would be no more than locations in *our* world, which would mean questioning of their status as separate worlds that are, in a sense, equal one with another, as well as with our world. Second, the theory of relations teaches us that the equivalence classes are disjoint sets. This would mean that possible worlds are also like this, that is, that they have no common

35 Richards (1975).

36 Every equivalence relation (the relation that satisfies the conditions of reflexivity, symmetry and transitivity) makes the equivalence classes on the set on which that relation is defined.

37 Davidson and Pargetter (1980, p. 390).

element. If such be the case, we would not be able to observe the same object through different worlds, nor the same state of things made by the relations between respectively the same objects and, consequently, we would not be able to contemplate the necessity of that state of things.

Similarly, we also encounter difficulty in Quine's idea of total mathematization of arbitrary world. All the states of things that we can locate physically and describe in a suitable space are recognized in accordance with that idea. Any determination of the matter's position can be precisely registered in a proper mathematical model, with discrete numeric values that make coordinates of the imagined coordinate system. However, we do not have clearly defined instructions how to recognize the states of things that do not have their obvious physical manifestation in that model. In this sense, the disputable thing is the mode of representing specific properties. Namely, within that model we could perhaps represent properties such as: *he wore a jacket with one missing button*, but we could not easily deal with the properties such as: *he was dejected due to a solitary life on an isolated island*. The latter group of properties include also the property *is positive*, which is important for us, being a property whose domain is again a defined set of features.[38] Indeed, by which n-tuples and in what coordinate system can the property of positiveness be represented? Should we regard it in a context of a specific argument, for example, how to represent a positive of justice, which is often mentioned in ethical and theological texts as a positive property?

Even though it is not easy to answer these questions, the reference books provide examples of attempts to save these concepts. One of them is based on the idea that a feature can also be understood as a type of the state of things and that, accordingly, it can be represented within a mathematical model with some additional modification of the described apparatus, regardless of the feature in question.[39] What kind of modification would that be? In Quine's construction, every point in space is represented by the ordered triples, the well-known methodology of analytic geometry. Generally speaking, we can discuss the ordered n-tuples, on the basis of which we can also determine the position in time, as well as abstract it from the three-dimensional

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[38] We use the word *properties* in these specific examples to mark, formally speaking, the unary predicate. It can be clearly concluded out of the Argument itself that the only domain that Gödel intended for the "is positive" predicate, and the only set on the basis of which he considered its interpretation, is the set of all properties.

[39] Heller (1998).

space. Now, with this supplementing modification, the whole space is described as a set of ordered pairs, where the first component is ordered n-tuple by which time-and-space positioning is carried out and the second is a set of numbers. Those numbers are, in effect, codes of primary properties [40] that are "distributed" on the position determined by the first component of the ordered pairs. Technically, it makes sense taking a set rather than an ordered n-tuples for the second component of the ordered pairs, because the sequence of features is not significant information on the location determined by the first component.[41] Therefore, we can speak also about representation of an arbitrary composite property (which is derived from the union of primary properties) in an arbitrary point in space. This entire construction appears technically natural and formally correct. However, the extension of n-dimensional methodology in description of the world implicitly suggests that only spatial-temporal and causally determinable states of things can be represented in this way. Representation of properties (both primary and complex) in points of space does not appear as intelligible analytic description, but rather as an attempt of artificial vivisection of something that cannot be represented in segments. This kind of description of properties would perhaps have some sense if it referred to the features that have their spatial and temporal manifestation, but this axiomatization cannot be applied to the properties of another kind.[42] For example, if φ_1 and φ_2 are two arbitrary "non-causal" properties, then it would be meaningless to talk about an ordered pair $((a_1, a_2, \dots , a_n), \{\varphi_1, \varphi_2\})$ as about actual representation of the properties φ_1 and φ_2 in the point $(a_1, a_2, \dots , a_n)$ of the n-dimensional space.

Kripke's notion of possible worlds appears to be most adequate for our purposes, that is, it is most suitable in helping us to accept the notion of necessity as mentioned in the Axiom. Kripke's worlds are not some distant countries into which we should travel in order to see what happens with the actual state of things from our world and, subsequently, be able to decide about the (non)necessity of the state of things. Nor are

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40 It is presumed that there is bijection between the set of primary features and the set of numbers representing those features.

41 It is not important whether we say that on a location determined by a specific n-tuple the features φ_1 and φ_2, or φ_2 and φ_1 are valid.

42 Unfortunately, the mentioned concept does not include any example of an actual feature, because it is not considered relevant (see Heller (1998)). It is clear, however, that not all features can be described in spatial and temporal terms.

they spaces that can be reached by means of technical devices of the modern civilization.

> "...But, once we see that such a situation is possible, then we are given that the man who might have lost the election or did lose the election in this possible world is Nixon, because that's part of the description of the world. "Possible worlds" are stipulated, not discovered by powerful telescopes."[43]

It could be said that possible worlds are images of possible variants of the actual states of things from our world, including the relevant conditions referring to those states. For example, we imagine Nixon in some other possible circumstances. One of the relevant conditions would include a support of a partner political party at the election. The idea of possible variants implies certain limits within which those variants can be observed. For instance, we can observe situations varying from the one in which he has an unreserved and complete support of the partner, through a situation in which that support is only partial, to the one in which he has no support whatsoever. Since we are interested to find out if the states of things/ properties are necessary, we therefore understand possible worlds as descriptions of possible conditions and variants entailed by those states of things/ properties. Some of those conditions have already been fixed and stipulated in all the theoretical variants that are possible for the state of things in question. If we consider some events related to Nixon, then his property *is man* has already been stipulated, it is invariable when considering any possible world, while it is by no means the case with the property *is married to a jealous woman*, which is variable, if at all relevant for his result at the election.

Embracing the previous interpretation, we can observe the properties in general in a similar manner. If the existence of an entity's property has not been stipulated, if it is not obligatory, but variable and dependable upon circumstances in which the entity is imagined, then the entity does not necessarily have that property. Conversely, if the idea of an entity entails inherent existence of a certain property of that entity, then we can say that it is a necessary property of that entity. We can observe the necessity in the Axiom in this sense as well. If an entity (the Argument states that it is an arbitrary feature from the domain of the unary predicate *is positive*) has a property of positiveness, then it does not mean that sometimes it may have it and

[43] Kripke (2001, p. 44).

sometimes it may not have it. In the Argument, positiveness is *stipulated* property of an object, if the object possesses it, regardless of the accidental structure of the worlds, that is, it possesses it in all the possible worlds. Whatever changeable circumstance we imagine in our world, if a property is positive in our world, whatever that implies, then it is positive in any of the possible worlds. Thus, positiveness is also a necessary property, since its invariability is stipulated through all the possible worlds. Intuitively, we may ask: why would positiveness, of all other properties, be stipulated in such a manner? An answer to this can be found in what is called the idea of proof in mathematics. In the Argument, God is imagined and defined in a natural way, that is, as the perfect being, the being that possesses all positive properties. If that is so, then it would be contra-intuitive and impractical for the argument to allow that positiveness is an impermanent and changeable property that depends on random circumstances in the world. It would not make sense to say that God possesses, for example, the property φ which is positive only in given circumstances, while in some other (other possible worlds) it is no longer positive.

Eventually, what is the connection between Kripke's idea of necessity and the notion of necessity we find in Gödel's axiom? According to Kripke, necessary states of things that refer to the specific objects/concepts imply, as already indicated, those states of things that can be found in all possible worlds. Therefore, in a certain sense, these are pre-supposed circumstances and presumably unchangeable when it comes to the specific objects in question. In other words, it is impossible to think about those objects without assuming those states of things. Those states of things are part of their nature, ascribed to them by "definition" and, thus, cannot be changed. For instance, we can return to the example from the number theory, which illustrates this, apparently being a good analogy of our case. The property *is prime*, implying the set of natural numbers as its domain, is also an example of a necessary property. Indeed, if a number is prime, then it is prime in all variable and accidental circumstances in which it can be observed or, to use the terminology of this text, it is prime in all the possible worlds. For instance, number 5 is necessarily prime. The very idea of the number 5 contains stipulated property (easily checked) pointing to the fact that it is a prime number. We can therefore draw an implication similar to that found in the Axiom: if a number is prime, then it is necessarily prime. In no other variable circumstances could number 5 be composite. Nevertheless, if such a case still somehow occurs, then it would no longer be number 5 as originally conceived, as the number which comes after 4 and before 6, as the number divisible only by 1 and by itself, etc. Thus, the property *is prime* has already been stipulated in its nature as the feature which does not depend on the accidental structure of the world. Likewise, the predicate is positive in Gödel's Ontological Argument is defined in such a manner that it does not depend on any random or changeable circumstances that can appear in the world. Gödel describes positivity as an attribute, which cannot depend on concrete circumstances featuring in the world. In other words, if a characteristic is positive, it is positive in all circumstances that can appear in the world. Obviously, if a characteristic is not positive, it is such in all circumstances occurring in the world, that is, in all possible worlds. Hence, of all the three analysed in this text, it is Kripke's idea of necessity, as based on the idea of possible worlds, that provides the best grounds for understanding the notion of necessity in the Ontological Argument. Kripke's understanding of necessity as something unchangeable, by definition ascribed to the object, is literally applied in Gödel's work. Positivity of a characteristic is a predicate, which the characteristic in question either has or does not have, and that predicate does not depend on random circumstances. Definition of an arbitrary characteristic implies definition of its (non)positivity. It is either positive or not, without depending on the circumstances occurring in the world. In Gödel's words: if a characteristic is positive, it is then positive by necessity. Thus, Kripke's understanding of the notion of necessity can be entirely incorporated into the intuitive background surrounding the notion of necessity in Gödel's Ontological Argument.

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